

*UNIVERSITY OF ELECTRONIC SCIENCE AND TECHNOLOGY OF CHINA*

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*DSP SEMESTER FINAL REPORT*

***Relationship among FT, DTFT, DFT, and z-transform***

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**Relationship between z-transform and DTFT**

Fourier transform (FT) is the major and is used widely. But it cannot be used for all systems. Sometimes FT does not exist for some particular reasons. FT types are Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT), and Discrete Time Fourier Transform (DTFT).

Z transform (ZT) on the other hand is used to simplify discrete time systems, e.g., digital signal processing, digital filter design, etc. It simplifies the analysis of the response of an LTI system to various signals and it can provide a means of characterizing an LTI systems and its response to various signals, by its pole-zeros locations.

The discrete-time Fourier transform (DTFT) or, simply, the Fourier transform of a discrete–time sequence x[n] is a representation of the sequence in terms of the complex exponential sequence {} where ω is the real frequency variable.

The discrete-time Fourier transform X() of a sequence x[n] is defined by

X()=

In general X( is a complex function of the real variable and can be written in rectangular form as

X()= Xre()+ jXim()

where Xre() and Xim() are, respectively, the real and imaginary parts of X(), and are real functions of ω .

The z transform of a discrete-time signal is defined as the power series

X(z)=

The DTFT is a special case of the z-transform. DTFT can be also expressed as.

X[k] = X(ω)|ω=(2**π/N**)k

We can write z-transform as

X[k]=X(z)|z=k

Z-transform characteristics:

* It is an infinite power series
* It exists only for those values of z, for which this series converges
* Region of convergence (ROC) of is the set of all values of z for which attains a finite value

Types of Z Transform

There are two types of z transform. The first one is bilateral z-transform which is two sided and we can describe it as:

X(z)=

The second type is unilateral z-transform which is single sided and we can describe it as follows:

X(z)=

The simplest relation between a finite-length sequence *x*[*n*]*,* defined for 0 n N-1, and its DTFT X() is obtained by uniformly sampling X() on the ω-axis between

0 ω 2π at ωk=2πk/N, for 0 k N-1.

Form X()=

X[k]= X(ω=2πk/N=, 0 n N-1.

The sequence *X*[*k*] is the discrete Fourier transform (DFT) of the sequence *x*[*n*].

**Relation between DTFT and DFT**

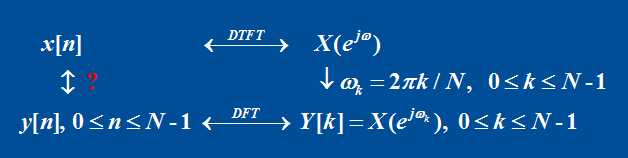
We can get DTFT from DFT by Interpolation.

We can express X() in terms of *X*[*k*] as follows.

X()= = ]

=

= .



The discrete-time Fourier transform (DTFT) and the z-transform are applicable to any arbitrary sequence, whereas the discrete Fourier transform (DFT) can be applied only to finite-length sequences.

**Relationship between FT and ZT**

The following two equations, Eq.(1) and (2) are FT and ZT, respectively.

X(ejω)=∑x[n]e-jω (1)

n=-∞  
 X(z)=∑x[n]z-n (2)

n=-∞  
Replacing z with ejω, ZTwill become FT

When ZT has unity magnitude (and FT exists), it means that ZT = FT. Then, FT is simply

X(z)=X(ejω) where z= ejω

If we evaluate the z-transform at http://www.elektroda.pl/cgi-bin/mimetex/mimetex.cgi?3$z%3De%5E%7B-j%5Comega%7D then we can get the DTF. This evaluation is equivalent to evaluating the z-transform on the circle in the complex plane.

ZT is possible to converge even if FT does not due to the sequence is multiplied by real exponential.

Fourier transform is concentrated and was originally made for continuous functions and z-transform works better than Fourier transform in discrete systems. It means that Fourier transform is for continuous systems whereas z-transform is for discrete systems.

Both Fourier transform and z-transform can convert discrete time domain to frequency spectrum domain. Both can give us frequency spectrum.

In DFT we can get discrete spectra from discrete input signals whereas in z-transform we can take discrete signal (input) and translate it into complex continuous function.

In addition to these z-transform is powerful for manipulating discrete signals.

DTFT and DFT are either specific or restricted versions of Fourier transform regarding to sampled signals or sampled signals with finite length and the z-transform is the generalizations of DTFT. DTFT is equivalent to the z-transform.

FT can be expressed as: Xa(F) =dt

= dF

DTFT is expressed as: x(ω) = =X(z)|z=

= dω

sampling sum of shifted replicates

x[n]=xa(nTs)

xa(t) x[n] xps[n]

bandlimited time limited

sinc interpolation rectangular wimdow

DTFS

FT DTFT DFT

Sum shifted scaled replicates sampling

Xa(F) x(ω) X[K]

Time-limited

bandlimited Dirichlet interpolation

rectangular window

unit circle sample unit circle

z

X(z)

One of the most important properties of the DTFT is the convolution property:

y[n] = h[n] \*x[n] DTFT H(w)X(w).

This property is useful for analyzing linear systems (and for filter design), and also useful for on paper convolutions of two sequences h[n] and x[n], since if the sequences are simple ones whose DTFTs are known or are easily determined, we can simply multiply the two transforms and then look up the inverse transform to get the convolution.

The discrete Fourier transform or DFT is the transform that deals with a finite discrete-time signal and a finite or discrete number of frequencies.

**Relation between DFT and z-Transforms**

The discrete Fourier transform or DFT is the transform that deals with a finite discrete-time signal and a finite or discrete number of frequencies.

By definition DFT can be written as:

X(k)= = |Ω=2πk/N

= X(Ω)|Ω=2πk/N

The DFT of *x*[*n*] is its DTFT evaluated at *N* equally spaced points in the range [0,2**π**). For a sequence for which both the DTFT and the z-transform exist, we can see that:

X(k)=X(z)|z=ej(2**πk/n)**

If x[n] is an L-point signal with L N then its DTFT will be

X[k] = X(ω)|ω=(2**π/N**)k

Its z-transform will be

X[k]=X(z)|z=k

By expressing X(z) in terms of X[k] we get another expression for z-tratnsform

X(z)=(1-z-N)/Nz-1

**Properties of Z- transform**

Since the z-transform is equivalent to the DTFT, the z-transform has many of the same properties. Specifically, the z- transform has the property of duality, and it also has a version of the convolution theorem. The z-transform is a linear operator.

**Convolution Theorem**

Multiplication in the discrete-time domain becomes convolution in the z-domain. Multiplication in the z-domain becomes convolution in the discrete-time domain.

**Stability**

It can be shown that for any system with a transfer function H(z), all the poles of H(z) must lie within the unit-circle on the z-plane for the system to be stable. Zeros of the transfer function may lie inside or outside the circle.

**Spectral analysis**

When the DFT is used for spectral analysis, the x{n} sequence usually represents a finite set of uniformly spaced time-samples of some signal x(t), where *t* represents time. The conversion from continuous time to samples (discrete-time) changes the underlying Fourier transform of x(t) into a discrete-time Fourier transform (DTFT), which generally entails a type of distortion called aliasing. Choice of an appropriate sample-rate (*Nyquist rate*) is the key to minimizing that distortion. Similarly, the conversion from a very long (or infinite) sequence to a manageable size entails a type of distortion called *leakage*, which is manifested as a loss of detail (aka resolution) in the DTFT. Choice of an appropriate sub-sequence length is the primary key to minimizing that effect. When the available data (and time to process it) is more than the amount needed to attain the desired frequency resolution, a standard technique is to perform multiple DFTs, for example to create a spectrogram. If the desired result is a power spectrum and noise or randomness is present in the data, averaging the magnitude components of the multiple DFTs is a useful procedure to reduce the variance of the spectrum (also called a periodogram); two examples of such techniques are the Welch method and the Bartlett method; the general subject of estimating the power spectrum of a noisy signal is called spectral estimation.

A final source of distortion (or perhaps *illusion*) is the DFT itself, because it is just a discrete sampling of the DTFT, which is a function of a continuous frequency domain. That can be mitigated by increasing the resolution of the DFT.

Fourier Transforms require complete knowledge of both Real and Imaginary parts of the magnitude and phase for all frequencies in the range –π < ω < π.

The Laplace transform is used to characterize and analyze signal and system interactions. If *x*(*t*) is the time domain signal, its Laplace transform is defined as:



Where *s* belongs to the set of complex numbers over which the integral converges.

If the Laplace transform is made discrete by sampling the time axis with interval *Ts*, it becomes:



Now let:



Substitute *z* to obtain:



Normalize sampling rate and define *Z*-transform as:



Where *z* belongs to the set of complex numbers for which summation converges. One-Sided Z-Transform Many applications assume the input starts at *t* = 0 (n=0 for discrete) and no response exists before t = 0. So the Z-transform is often written as**:**



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